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PROJECT MANAGEMENT SUBJECT TO IMPRECISE ACTIVITY NETWORK AND COST ESTIMATION CONSTRAINTS

Abstract

The new approach to project planning assuming soft links between activities and imprecise cost of activities execution is considered. In that context, the method allowing one to estimate the duration and the cost of project execution is proposed. The illustrative example emphasizing the advantages of the approach proposed is enclosed.

1. INTRODUCTION

The inside and outside conditions of contemporary enterprise are full of uncertainty caused of changing customer requirements, economical turbulence, resource utilization, personnel mobility and their creativity. Under such conditions the enterprises have to manage several different projects overlapping in time as well as using the same set of resources. In that context such imprecise data as duration time of activities, cost of each activity, utilisation rate of resources, and so on, have to be taken into account, while responding to the growing needs to solve uncertainty in project planning [1]. The disturbances occurring usually lead either to project makespan extension or to project's budget escalation or to both of them simultaneously. Such cases may be avoided, however under assumption that many alternative scenarios of project execution are enabled. It means that allowing different alternative precedence relationships in project's activity network as well as alternative resources for activities execution may result in more than one possible variant of project execution.

Therefore, special techniques are needed to provide a simple way of project planning updating in order to reflect the impact of change on project duration and project cost. The soft logic [10] allowing considering more than one possible sequence of activities in activity network follows these expectations. Such model usually provides more than one feasible variants of considered project execution. Each alternative can be then evaluated from the point of view of its duration and execution cost.

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In this paper the model combining the precise (crisp) and imprecise (fuzzy) values of decision variables as well as hard and soft arcs of activity network are considered. Such model allows considering the possibility of "weakening" of relations in activity network and imprecise character of operation times activities on duration and cost of project execution while providing opportunities similar to the concept of agile project management.

The rest of the paper is organized as follows: Section 2 provides the problem statement. An approach to modelling of soft links between activities and imprecise cost of activities execution aimed at estimation of duration and cost of project execution is presented in Section 3. In Section 4, an illustrative example showing advantages following from the approach proposed is discussed. Conclusions are presented in Section 5.

2. PROBLEM STATEMENT

Consider project containing a set of "n" independent activities. The project is represented by an activity-on-node network. That is assumed, the duration time of each activity is represented as a crisp value as well as each activity cannot be preempted. The relations between activities can be hard and soft. Each variant of project activity network results in possible variant of project execution. That means from the one side activities share and compete for the same resources, from the other side however only a part of the resources can be used for the project's execution.

That means a project can be executed in different alternative ways. Such alternative ways can be seen as opportunity to react to changes of external factors such as recession on the market, limited resources, and changes of clients' requirements. Therefore the precedence relations in activity network could be adjusted due to changed occurred as to be more appropriate to the new conditions.

The changes in the activity network can delay or accelerate the project duration time. Moreover the updating of activity network can lead to total cost reduction or increase of project. So, the changes in the project execution can lead either to better or worse resources exploitation. Therefore, in order to obtain alternative variants of project execution some changes in the soft links of activity network should be introduced. Of course, the results of changes in project network model should be evaluated.

Let us assume the activities cost are uncertain and specified by a fuzzy numbers estimated by experts. The problem considered regards of a feasible activity network that follow the constraints imposed by the hard and soft precedence relations. The objective is to find such a solution which guarantees the project duration and project cost of project execution satisfy assumed values.

The approach proposed seems to be attractive alternative to the traditional one concerning only to unique scenario of project execution. A lot of variants can ensure the best available solution from the considered possibilities. That is because assuming searching among alternative project scenarios provides the framework allowing one to deal with the project management uncertainty.

3. MODELLING

3.1. Precedence relationship in activity network

In practice some sequences between activities in activity network may be changeable or soft. Therefore, special techniques are needed to provide a simple way of network updating in order to reflect the impact of logic change on project completion date and on the critical path [10].

Network models of undetermined logical structure of Generalized Analysis Network (GAN) provide possibility of multi-variant way to establish relations between activities and creative selection of other proceedings during realization of undertaking than previously arranged [7]. The concept of weak relations stems from the network planning method of Graphical Evaluation and Review Technique (GERT). The GERT method enables us to display the course of the project and its possible variants in the form of stochastic network by inserting into the structure of the network of alternative sequences of possible variants of operations, also all activities, even those of little possibility to arise and search for the course of the project according to one of the possible scenarios.

Alternative-ruling networks enable modelling of various variants of accomplishment of projects with variable structure and scope. Relations between activities may be of both “strong” and “weak” character. For example, one may not place a concrete mix without prior boarding and reinforcing of the construction (type of relation: "hard"). This kind of relation cannot be changed. However a relation: “starting to make prime coat after making plaster” may be reverted. It is an example of a “weak" relation.

A modification of the Critical Path Method (CPM) resulting in an algorithmic procedure SOFTCPM has been developed to handle the soft logic in network analysis [10]. SOFTCPM is a microcomputer program deals with the soft logic in CPM networks. It has the capability of updating the CPM network logic when any unexpected event occurs that prevents working according to the scheduled activity sequence.

Another work [12] contains examples of using the PROSOFT method, which is based on three types of weak relations defined by El-sersy. To present soft logic in a project network model the following types of relations are used:

- OR relation – means that two activities linked by OR relation may be performed one after another, i.e. after the previous activity was ended; the other permissible possibility is parallel performance of both activities, OR relation is marked with a circle on an arc,
- EXCLUSIVE-OR relation – means, that reverse sequence of activities belonging to the same type of relation is permissible, the symbol of EXCLUSIVE-OR relation is marked by two vertical lines on an arc.
- SOFT relation – permits three types of performing given activities, i.e. one after another, one after another but in a reverse sequence or parallel, SOFT sequence is marked with a broken line on an arc.

Figure 1 presents highlighted types of sequence relations. The OR relation between activity X and activity Y denotes that activity X can precede acitivity Y or the activities can be performed simultaneously. The second alternative sequences means that the relation between activities is ignored. The EXCLUSIVE-OR arc from activity X to activity Y indicates that activity Y can precede activity X, but both activities cannot be executed simultaneously. The last type of not fixed relations SOFT arc from activity X to activity Y implies that activity Y can precede activity X or the activities can be executed simultaneously.

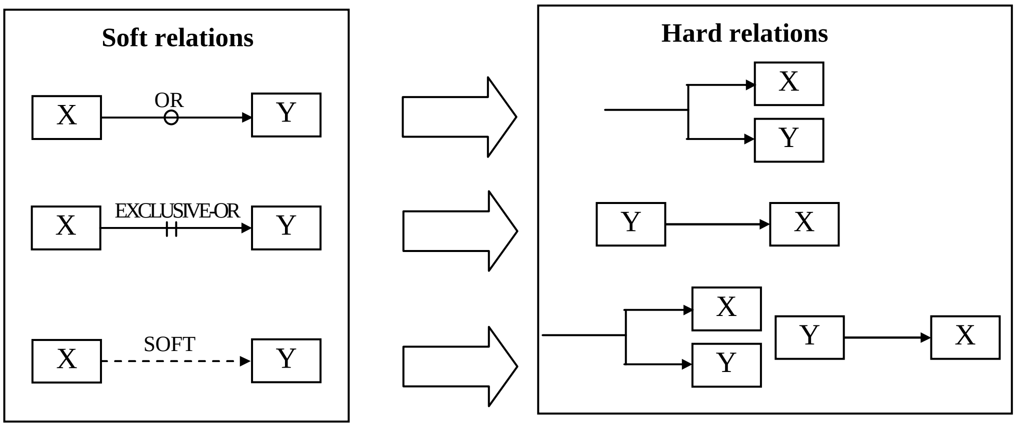

Fig. 1. Hard and soft relations [12]

An example of an activity network with highlighted hard relations and a single soft relation is shown in Fig. 2. Introduction of the SOFT type relation leads to creation of two alternative variants of activity networks, see Fig. 3, and Fig. 4. The alternative variants enlarge the amount of possible ways to perform a given project. Since the choice of the way the project is accomplished influences the total time of project realization and its cost, hence specification of basic parameters of particular variants before choosing the way of project execution is of crucial importance.

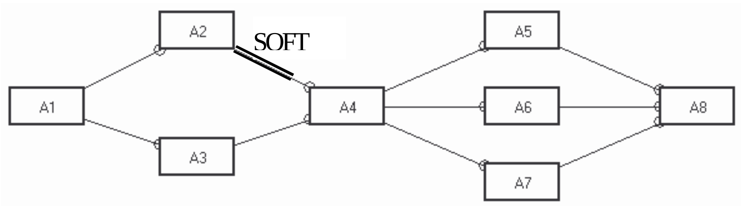

Fig. 2. Network project model with SOFT arc

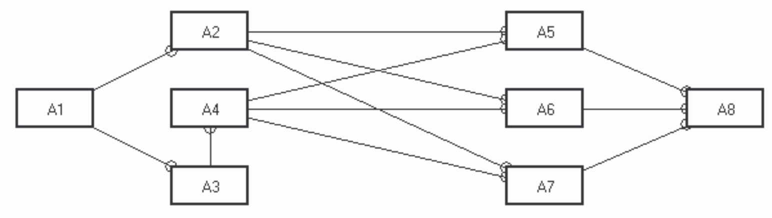

Fig. 3. Network project model after ignoring the A2 $\prec$ A4 arc

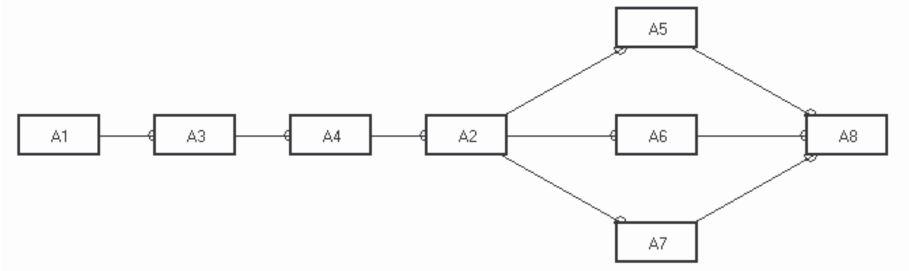

Fig. 4. Change in network project model due to reversing the A2 ≺ A4 arc

3.2. Cost estimation

Cost management includes among others: determining resources necessary to accomplish the project, estimation of given resources in quantities needed to accomplish the project, budgeting and control over changes in the budget. The objective of cost management is to guarantee accomplishment of the project according to intended budget. Project cost estimation, so fundamental to all project, is determining of predicted costs of use of resources engaged in accomplishment of the given project. Due to R. Keeling, the set of resources engaged into accomplishment of projects includes [11]:

1. Money.
2. Material.
3. Merchandise.
4. Machinery.
5. Manpower.
6. Management, professionals and specialists.
7. Movement.

Since the availability of information during the early stages of a project is quite limited, since managers leverage their knowledge, experience and estimators to estimate project costs. In other words, they usually rely on their intuition. In practice there are many methods of estimation of costs of project accomplishment. Methods used to set project costs may be divided into three groups [9]:

1. Approximation methods.
   - assessing by experts,
   - conscious guessing,
   - rough estimation,
   - empirical,
   - by analogy.
2. Parametrical methods:
   - cost estimation equations,
   - statistical,
   - probabilistic,
   - mathematical models.
3. Detailed methods:
   - on the basis of job batches,
   - on the basis of work preparation methods,
   - other detailed methods.

That should be noted however that methods of above mentiond kind may be helpfull in achieving general estimates of project cost, but may fail facing with uncertainty. The question arising is: How to perform estimation of uncertain project cost?

So, taking into account the disadvantage the existing methods of cost estimation suffer from, a new concept of project cost estimation is proposed, i.e. the concept which takes into consideration uncertainty in assessment of costs while using fuzzy sets. Of course, because fuzzy numbers are used to express the cost of all activities in the project, the total project cost is also a fuzzy number. Short introduction to the fuzzy set formalism employed in the paper is available in the APPENDIX attached.

3.3 Decision making

The fuzzy decision making procedure consists of following elements: fuzzification, fuzzy database rules, fuzzy decision making, and defuzzification, see Fig. 5.

Fuzzification stage: At this stage the measured value of the input variable is converted in to the fuzzy set of variables. This operation is called fuzzification. The fuzzification is done by giving values to each of a set of a membership function. The value for each fuzzy number is determined by the variable and the shape of membership function [17]. The triangular membership function is the simplest one and gives good results. An example of the triangular membership function was shown on Fig. 20 in an APPENDIX.

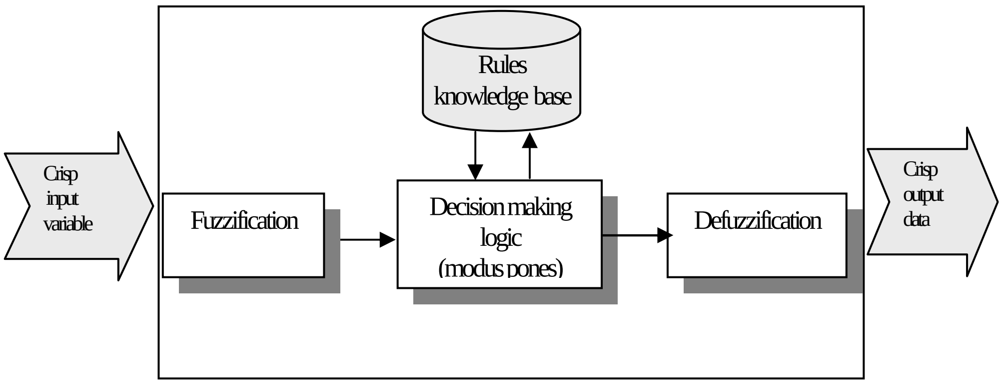

Fig. 5. The structure of fuzzy decision making system

Fuzzy database rules: The fuzzy database rules consist a set of fuzzy rules of the following form "**If-Then**-". Variables may adopt both values estimated in words, e.g. "small", "medium", "high", and numerical values. Single rule in the base of rules may have the following form:

IF {Project Time is Medium} **AND** {Project Cost is Low} **THEN** {Attractiveness of the Project is Medium}

Rules are created by experts on the basis of knowledge possessed by them and experience acquired in the given field.

Fuzzy decision stage: The decision making is similar to simulating human decision making in inferring fuzzy control actions based on the fuzzy rules of inference in fuzzy logic.

At this stage the knowledge base and implemented methods are used to solve the given problem. An example of inference mechanism based on the rule with two inputs and one output so called Min Inferencing, is presented in Fig. 6. The entire strength of the rule is considered as the minimum membership value of the input variables' membership values. In case of larger number of rules in the knowledge base, obtained membership functions for all rules are aggregated, in order to achieve final function of membership.

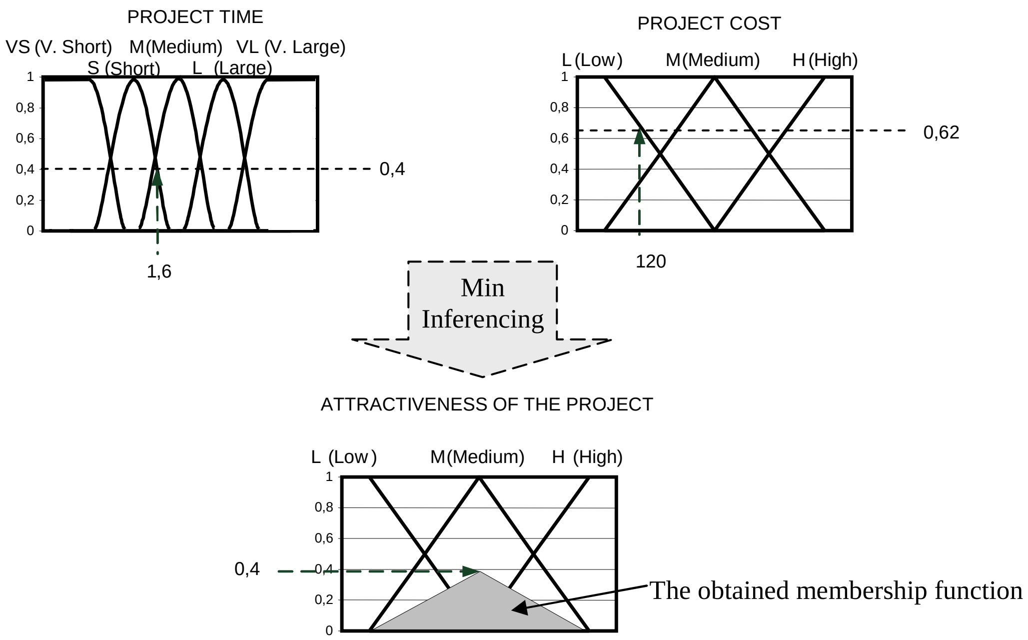

Fig. 6. Implementation of fuzzy rule set

Defuzzification stage: At this stage the fuzzy variables generated by the fuzzy logic rules are turned into a non-fuzzy value. The fuzzy logic process which does this is called defuzzifacation because it combines the fuzzy inputs to give a corresponding real (non-fuzzy or crisp) output which can be used to perform some action. The defuzzifier combines the information in the fuzzy inputs to obtain a single crisp (non-fuzzy) output variable. Among many methods of defuzzification, the most common one is the Centre of Gravity Method (COG), see formulae (4) [6, 17].

$$y^{*} = \frac{\sum_{i=1}^{m} y_i \cdot \mu_{wyn_i}}{\sum_{i=1}^{m} \mu_{wyn_i}} \tag{1}$$

where:

y_i – *i*-th value of output variable,

μ_{wyn_i} – value of obtained membership function for *i*-th value of output variable,

m – a number of discrete values of output variable.

4. ILLUSTRATIVE EXAMPLE

There exists a project represented by the activity network G = (V, E), where: V - is a set of nodes, and E - is a set of arcs. As an example, we consider the A-O-N network depicted in Fig. 7. Nodes of a network are assigned to the activities. The project network consists of 15 activities. The arcs in the project represent precedence relations. According to precedence relations, the relations in the set of activities can be fixed or soft. In the project network OR, SOFT and EXCLUSIVE-OR soft relations types are used.

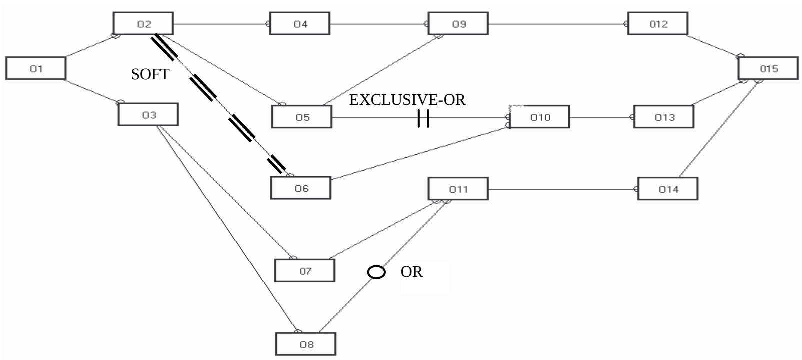

Fig. 7. Precedence relations for the project

Initial analysis of the project network model of the discussed project shows, that the sequence of performing some activities is precisely determined, whereas other activities may be performed in other way. Variable character of sequential relations are of hard or soft character.

4.1 Project duration

The case the activity O1 precedes the activity O2 is denoted by: O1 $\prec$ O2. It means, that the sequence of performance of particular activities is precisely determined and may not be changed. Another example of hard relations is the relationship between activities: O2 $\prec$ O5, O2 $\prec$ O4, O5 $\prec$ O9, O6 $\prec$ O10, O9 $\prec$ O12, O1 $\prec$ O3, O3 $\prec$ O7, O3 $\prec$ O8, O7 $\prec$ O11, O11 $\prec$ O14, O10 $\prec$ O13, O12 $\prec$ O15.

Two activities can begin at the same point in the time, for example O4 ad O5, in symbols O4 ~ O5. Apart from hard relations there are three types of soft arcs: OR, EXCLUSIVE-OR, SOFT. Soft ars allow different way of realization of activities.

Introducing three soft relations leads to creation of alternative, acceptable variants of accomplishment of the discussed project, see Fig. 8. OR and EXCLUSIVE-OR relation generate one alternative respectively, whereas SOFT relation – two alternatives. Activity O8 may be performed before activity O11 or parallel with activity O11 (OR relation).

EXCLUSIVE-OR relation between activities O5 and O10 means, that activity O5 may be performed in a set sequence, i.e. before finishing activity O10 or in a reversed sequence, i.e. activity O10 may be performed directly before activity O5. Connecting activities O2 and O6 by SOFT relation means, that given activities may be accomplished in the adopted sequence $O2 \prec O6$, in a reversed sequence $O6 \prec O2$ or both of those activities may be performed parallel.

Taking into soft arc from Fig. 7 leads to modification of activity network considered (see Fig. 8).

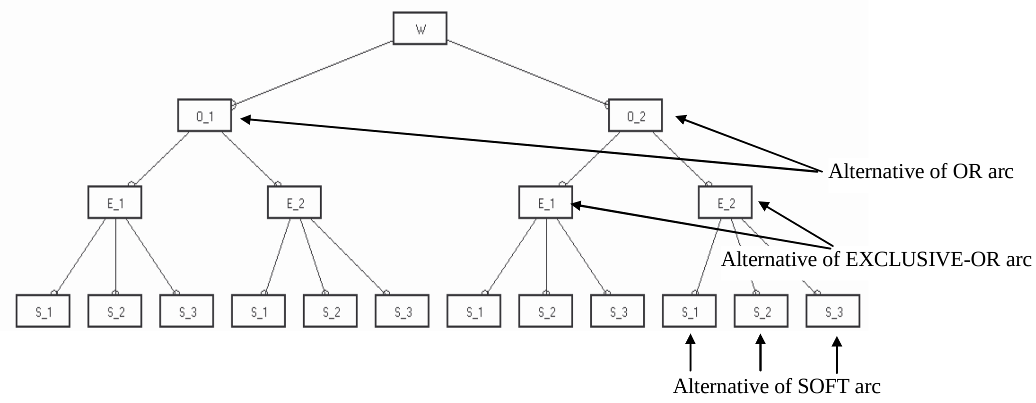

Fig. 8. The tree of searching a feasible variants of project network model

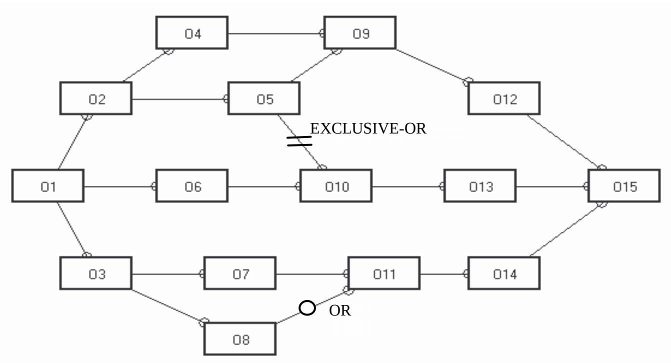

Fig. 9. Feasible vaiant of project network model after activating SOFT arc

One of the admissible scenarios of the project network model is shown in Fig. 9. It results from it, that activities O2 and O6 may be performed parallel due to taking into consideration SOFT link. Remaining relations between activities were unchanged.

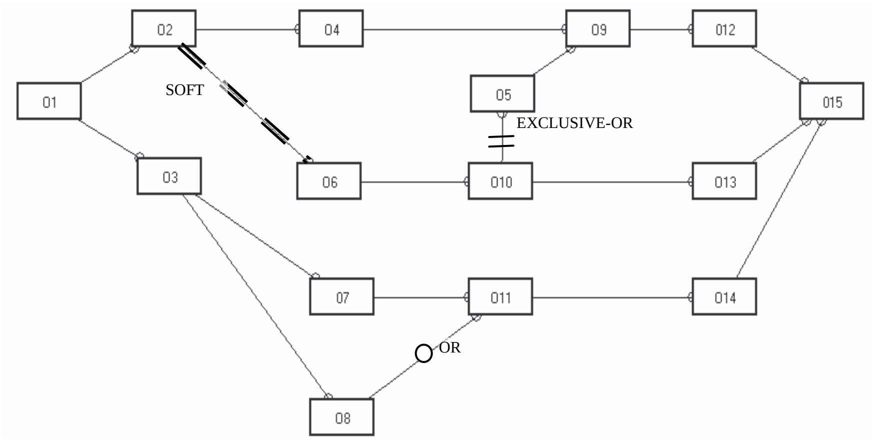

Fig. 10. Feasible vaiant of project network model after activating EXCLUSIVE-OR arc

Figure 10 presents feasible variant of the project created after EXCLUSIVE-OR arc was activated. It results from it, that activity O5 is performed in the reverse sequence, i.e. directly after activity O10 was ended: $O10 \prec O5$, activity O2 and activity O6 may be performed in sequence $O2 \prec O6$. Sequence of realization of other activities remained unchanged.

Another variant (Fig. 11) indicates, that the considered project is feasible in different set of sequence of activities. Activities O2 and O6 may be performed simultaneusly (just like in Fig. 9) with inverse performance sequence of activities O5 and O10.

Figure 12 is a graphical illustration of permissible variant of accomplishment of the project where O8 and O11 activities may be performed simultaneusly, OR relation was taken into account in this case.

In order to illustrate influence of soft relations on total time of the considered project, net models were determined for each of the five presented variants, as well as critical paths and planned time for performance of the project in each sequential system. It was assumed, that duration of activities is known and presented by a single value – deterministic approach. Table 1 presents duration time for each particular activities in the conventional time units (c.t.u.).

Table 2 presents determined critical paths of particular variants of project performance and quantity of total time of project performance. It results from it, that the shortest duration of realization of the considered project amounts to 28 conventional units of time. Performance of the project in variant first, second and fifth leads to performance of the project in the shortest time. Adopting minimization criterion of project duration time, first, second and fifth variants are optimal. Variant No 4, which takes into consideration SOFT relation (simultaneously performance of activities O2 and O6) and EXCLUSIVE-OR link (reversed sequence of activities O5 and O10), leads to prolonging time of project by 8 conventional time units up to 36 c.t.u.The last variant presented in the work (variant No 3) is characterized by the longest time of performing given project. In this case the total time of project performance is 38 c.t.u.

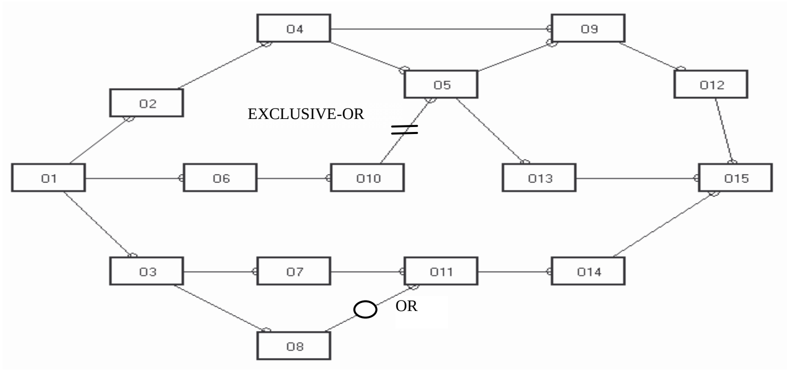

Fig. 11. Feasible vaiant of project network model after activating SOFT i EXCLUSIVE-OR arcs

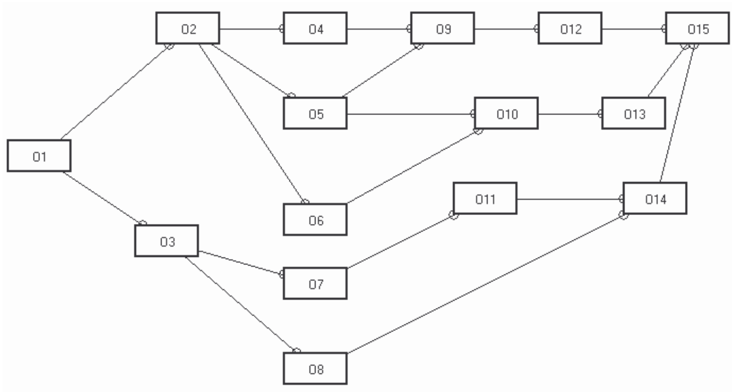

Fig. 12. Feasible variant of project network model after activating OR arc

Tab. 1. Duration time for each activity in project network model

Number of activity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Duration time[c.t.u.]	**3**	**2**	**3**	**1**	**5**	**6**	**4**	**2**	**8**	**4**	**3**	**7**	**5**	**8**	**3**

4.2 Project cost

Subsequent stage of project variants analysis includes cost estimation. Costs of accomplishment of activities of the project are modelled with the use of triangle fuzzy numbers. Three experts joined the process of cost estimation. Their task is to estimate value of particular works. The costs of performance of an activity estimated by the experts were averaged. Averaged triangle numbers representing average costs of activities were subsequently subjected to defuzzification. As a result of defuzzification the value of real cost of performance of given activities was obtained and then the value of real cost of the considered project.

Tab. 2. Comparison list of results of particular scenarios of project performance

Number of project variant	**Number of figure project network model**	**Crital path**	**Total time of project [c.t.u.]**
1	Fig. 7	$O1 \prec O2 \prec O5 \prec O9 \prec O12 \prec O15$	28
2	Fig. 9	$O1 \prec O2 \prec O5 \prec O9 \prec O12 \prec O15$	28
3	Fig. 10	$O1 \prec O2 \prec O6 \prec O10 \prec O5 \prec O9 \prec O12 \prec O15$	38
4	Fig. 11	$O1 \prec O6 \prec O10 \prec O5 \prec O9 \prec O12 \prec O15$	36
5	Fig. 12	$O1 \prec O2 \prec O5 \prec O9 \prec O12 \prec O15$	28

Number 1 of project variant is considered at stage of project cost estimation. In Table 3 assessments presented by experts were included, which determine performance cost of particular activities of the project in conventional cost units (c.c.u.). The table also includes comparison list of averaged performance costs of particular activities. The last column in the table concerns real cost of given project variant determined with the use of chosen method of defuzzification.

Tab. 3. List of project cost

No of activity	**Cost of activity estimated by expert [c.c.u.]**			**Averaged cost [c.c.u.]**	**Real cost [c.c.u.]**
	Expert 1	**Expert 2**	**Expert 3**		
	(l_1, m_1, u_1)	(l_2, m_2, u_2)	(l_3, m_3, u_3)	$(\boldsymbol{l_{average}}; \boldsymbol{m_{average}}; \boldsymbol{u_{average}})$	$\mu_{\tilde{K}_{DEF}}$
1	(2, 5, 7)	(3, 4, 6)	(2, 4, 5)	**(2,33; 4,33; 6)**	**4,22**
2	(3, 4, 5)	(2, 3, 5)	(2, 3, 4)	**(2,33; 3,33; 4,67)**	**3,44**
3	(10, 12, 14)	(8, 9, 10)	(9, 10, 11)	**(9; 10,33; 11,67)**	**10,33**
4	(1, 2, 3)	(1, 2, 3)	(2, 3, 4)	**(1,33; 2,33; 3,33)**	**2,33**
5	(5, 6, 7)	(4, 6, 7)	(5, 6, 7)	**(4,67; 6; 7)**	**5,89**
6	(4, 5, 6)	(4, 6, 7)	(5, 6, 7)	**(4,33; 5,67; 6,67)**	**5,56**
7	(8, 10, 11)	(9, 11, 12)	(10, 11, 12)	**(9; 10,67, 11,67)**	**10,44**
8	(6, 7, 8)	(7, 8, 9)	(8, 9, 10)	**(7; 8; 9)**	**8**
9	(11, 12, 13)	(10, 11, 12)	(10, 12, 13)	**(10,33; 11,67; 12,67)**	**11,56**
10	(7, 8, 9)	(8, 9, 10)	(8, 9, 10)	**(7,67; 8,67; 9,67)**	**8,67**
11	(3, 4, 5)	(5, 6, 7)	(5, 7, 8)	**(4,33; 5,67; 6,67)**	**5,56**
12	(3, 4, 5)	(2, 3, 5)	(2, 3, 4)	**(2,33; 3,33; 4,67)**	**3,44**
13	(12, 14, 15)	(13, 14, 15)	(12, 13, 15)	**(12,33; 13,67; 15)**	**13,67**
14	(14, 15, 17)	(16, 17, 18)	(14, 16, 18)	**(14,67; 16; 17,67)**	**16,11**
15	(1, 2, 3)	(1, 2, 3)	(2, 3, 4)	**(1,33; 2,33; 3,33)**	**2,33**
			Sum	**(92,98; 112; 129,67)**	**111,55**

The cost of realization of the given project in variant one is provided with a triangular fuzzy number (92,98; 112; 129,67) c.c.u. The single value of the cost of project realization is 111,55 c.c.u. The cost does not deduct other values resulting from taking soft relations into account.

Changed order of realization of particular activities implies the necessity of additional work. The work increases the demand for financial means. Individual experts have estimated the cost of additional work with the use of triangular fuzzy numbers. It has been assumed that a change of a character of a relation by considering a weak relation of the OR, EXCLUSIVE-OR, SOFT type results in the increase of costs. Costs of additional jobs provided in c.c.u. are presented in Tab. 4.

Tab. 4. Costs of additional jobs

Relation type		Cost of activity estimated by expert [c.c.u.]			Averaged cost [c.c.t.]	Real cost [c.c.u.]
		Expert 1	Expert 2	Expert 3		
		(l_1, m_1, u_1)	(l_2, m_2, u_2)	(l_3, m_3, u_3)	$(l_{average;}\ m_{average;},\ u_{average})$	$\mu_{\tilde{K}_{DEF}}$
OR simultaneously		(2, 5, 6)	(3, 5, 6)	(4, 5, 6)	**(3; 5; 6)**	**4,67**
EXCLUSIVE-OR reversing		(3, 4, 5)	(3, 4, 5)	(3, 4, 5)	**(3; 4; 5)**	**4**
SOFT	simultaneously	(1, 2, 4)	(3, 4, 5)	(2, 3, 6)	**(2; 3; 5)**	**3,33**
	reversing	(4, 5, 6)	(3, 4, 5)	(2, 3, 4)	**(3; 4; 5)**	**4**

4.3 Project selection

In relation to the above-mentioned variants of realization of a given project resulting from the use of defined soft relations, feasible costs of projects realization have been determined. The value of costs is listed in Tab. 5. The table includes relevant alternative variants the completion times.

Tab. 5. Total time and total cost project variant

Number of variant	Total time [c.t.u.]	Total cost [c.c.u.]
1	28	**(92,98; 112; 129,67)**
2	28	**(94,98; 115; 134,67)**
3	38	**(95,98; 116; 134,67)**
4	36	**(97,98; 119; 139,67)**
5	28	**(95,98; 117; 135,67)**

In the first variant the cost of project realization is the smallest and equals appropriately (92,98; 112; 129,67) c.c.u. The second variant is a bit more expensive; in this case the span of the cost of a project is (94,98; 115, 134,67) c.c.u. The next variant is characterized by a relatively low cost (95,98; 116; 134,67) c.c.u. and the longest time of project duration, which is 38 c.t.u. The fourth variant is the most expensive. It has the highest cost of project completion.

The cost is the result of considering the possibility of soft link and equals (97,98; 119; 139,67) c.c.u. The fifth variant is characterized by the shortest time of project realization. The cost of project realization in this variant is a little higher than the cost in a variant with the lowest cost and equals (95,98; 117, 135,67) c.c.u.

The determined variants of project realization will undergo complete evaluation with the use of the fuzzy system implemented in Matlab software package. Two input variables are considered: time of project variant realization (TofPVR) and cost of project variant realization (CofPVR). The output variable is only one, i.e. Attractiveness of project variant realization (AofPVR). The value of output variable is determined from the aggregation of impact two input variable.

The input and output variables and their membership function are suggested by experts. The experts defined the fuzzy numbers. They used the triangular fuzzy numbers, because these kind of fuzzy numbers are easy to implement and give good results. The variables were split into five subset to fuzzify as follows:

1. VL – time/cost/attractiveness of project variant realization is Very Low.
2. L – time/cost/attractiveness of project variant realization is Low.
3. M – time/cost/attractiveness of project variant realization is Medium.
4. H – time/cost/attractiveness of project variant realization is High.
5. VH – time/cost/attractiveness of project variant realization is Very High.

The Fig. 13, and Fig. 14 present the fuzzification of the input variables. In turn the fuzzification of the output variable is shown in Fig. 15.

A fuzzy decision making system is obtained by writing a set of rules. The rules are based on experts' experience. Cause-and-effect dependencies of attractiveness of project variant realization have been based on the pairs: time of project variant realization – cost of project variant realization. Such a choice of knowledge base is dictated by the ease of intuitive determination of fuzzy inference results. The combined impact of the TofPVR ad CofPVR on attractiveness of project variant realization is evaluated, according to a fuzzy decision rule (Tab. 6).

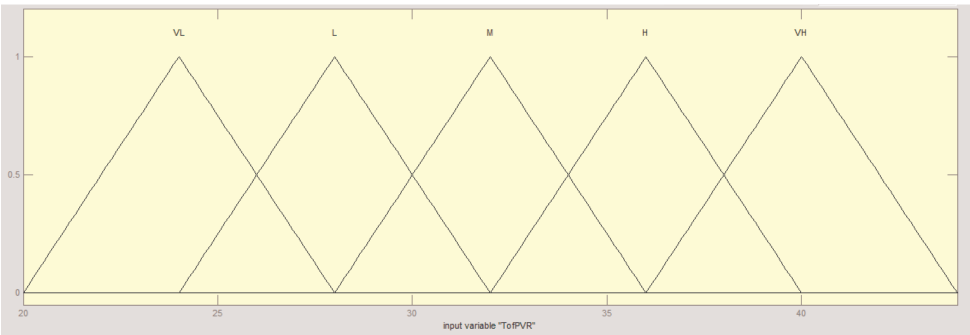

Fig. 13. The complete set of fuzzy of membership function of time of project variant realization

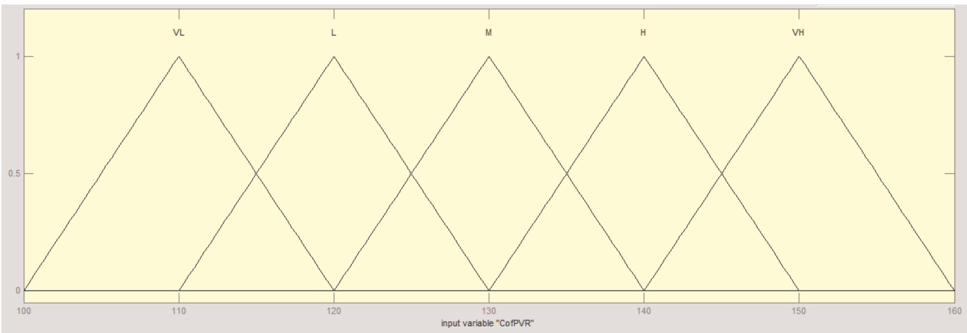

Fig. 14. The complete set of fuzzy of membership function of cost of project variant realization

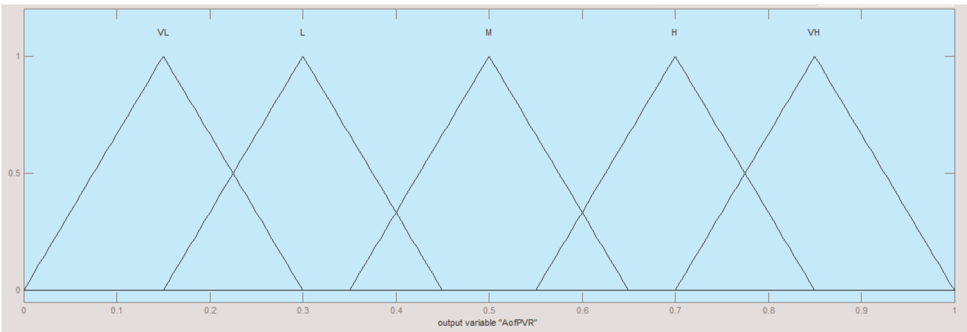

Fig. 15. The complete set of fuzzy of membership function for five levels fuzzifications of ouput variable

Tab. 6. The knowledge base

Attractiveness of project variant realization (AofPVR)						
Time of project variant realization (TofPVR)		**VL**	**L**	**M**	**H**	**VH**
Cost of project variant realization (CofPVR)	**VL**	**VH**	**VH**	**H**	**M**	**L**
	L	**VH**	**H**	**H**	**M**	**L**
	M	**H**	**H**	**M**	**L**	**L**
	H	**M**	**M**	**L**	**L**	**VL**
	VH	**M**	**L**	**L**	**VL**	**VL**

Taking into account the above mentioned dependencies, the system knowledge base consists of twenty five conditional complex rules, where there are two elements of the rule combined by the logical conjunction AND in the antecedent. Some examples of these IF-THEN rules are presented below.

IF {TofPVR VL} **AND** {CofPVR M} **THEN** {AofPVR H}
IF {TofPVR L} **AND** {CofPVR H} **THEN** {AofPVR M}
IF {TofPVR H} **AND** {CofPVR VH} **THEN** {AofPVR VL}

Fig. 16 presents an example of using fuzzy rule as follow:

IF{TofPVR L} **AND** {CofPVR L} **THEN** {AofPVR H}

Fig. 17 illustrates a complete set of fuzzy rules implemanted in the application. The evaluation of a fuzzy rule is based on computing the truth value of its antecedent and applying it to its consequent. This results in assigning one fuzzy subset to each output variable true. In Min Inteferencing where parts of fuzzy rules are labelled with AND logical operation then the fuzzy AND is obtained as the minimum of the membership values of the input variables' membership values.

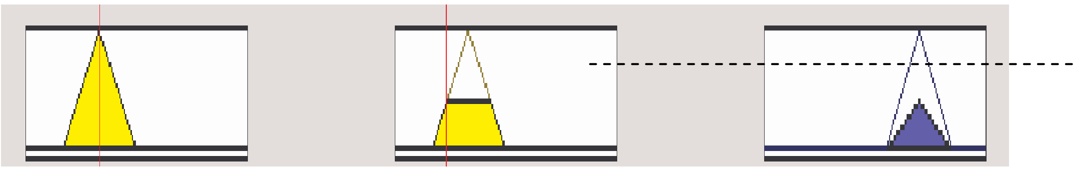

Fig. 16. An example of using fuzzy rule

1. (TofPVR==VL) & (CofPVR==VL) => (AofPVR=VH) (1)
2. (TofPVR==VL) & (CofPVR==L) => (AofPVR=VH) (1)
3. (TofPVR==VL) & (CofPVR==M) => (AofPVR=H) (1)
4. (TofPVR==VL) & (CofPVR==H) => (AofPVR=M) (1)
5. (TofPVR==VL) & (CofPVR==VH) => (AofPVR=L) (1)
6. (TofPVR==L) & (CofPVR==VL) => (AofPVR=VH) (1)
7. (TofPVR==L) & (CofPVR==L) => (AofPVR=H) (1)
8. (TofPVR==L) & (CofPVR==M) => (AofPVR=H) (1)
9. (TofPVR==L) & (CofPVR==H) => (AofPVR=M) (1)
10. (TofPVR==L) & (CofPVR==VH) => (AofPVR=L) (1)
11. (TofPVR==M) & (CofPVR==VL) => (AofPVR=H) (1)
12. (TofPVR==M) & (CofPVR==L) => (AofPVR=H) (1)
13. (TofPVR==M) & (CofPVR==M) => (AofPVR=M) (1)
14. (TofPVR==M) & (CofPVR==H) => (AofPVR=L) (1)
15. (TofPVR==M) & (CofPVR==VH) => (AofPVR=L) (1)
16. (TofPVR==H) & (CofPVR==VL) => (AofPVR=M) (1)
17. (TofPVR==H) & (CofPVR==L) => (AofPVR=M) (1)
18. (TofPVR==H) & (CofPVR==M) => (AofPVR=L) (1)
19. (TofPVR==H) & (CofPVR==H) => (AofPVR=L) (1)
20. (TofPVR==H) & (CofPVR==VH) => (AofPVR=VL) (1)
21. (TofPVR==VH) & (CofPVR==VL) => (AofPVR=M) (1)
22. (TofPVR==VH) & (CofPVR==L) => (AofPVR=L) (1)
23. (TofPVR==VH) & (CofPVR==M) => (AofPVR=L) (1)
24. (TofPVR==VH) & (CofPVR==H) => (AofPVR=VL) (1)
25. (TofPVR==VH) & (CofPVR==VH) => (AofPVR=VL) (1)

Fig. 17. The set of fuzzy rules implemented.

The last stage in building fuzzy making decision system is defuzzification. The output variables are defuzzified to get a crisp value. The defuzzification employs the centre of gravity method, which is illustrated in Fig. 18.

The following variants of the project realization have been evaluated by the developed system. The calculations made for five project realization variants are presented in Table 8. Single, crisp marks of particular variants are different.

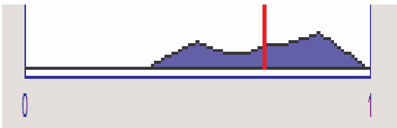

Fig. 18. The illustration of center of gravity method

The first variant is characterized by the shortest time and the lowest cost. The variant received the highest mark, which is 0,821. Increasing project cost by 3 c.t.u. and assuming unchanged duration time provides the second realization variant of the project considered. The note is lower and equals 0,775. A similar note was given to the fifth variant, in which a OR link was activated. As a result of the process duration time of the project was still the shortest. What increased was the variant realization cost, which was 117 c.c.u.

Significant extension of project realization time in variants three and four results in the lowest notes, accordingly 0,432 and 0,5. The two variants are the least desirable in project realization. The inference mechanism for variant one is presented in Fig. 19.

Tab. 7. The global evaluation of project variants

Number of variant	**Total time [c.t.u.]**	**Total cost with α=1 confidence level [c.c.u.]**	**Crisp value**
1	28	112	0,821
2	28	115	0,775
3	38	116	0,432
4	36	119	0,500
5	28	117	0,744

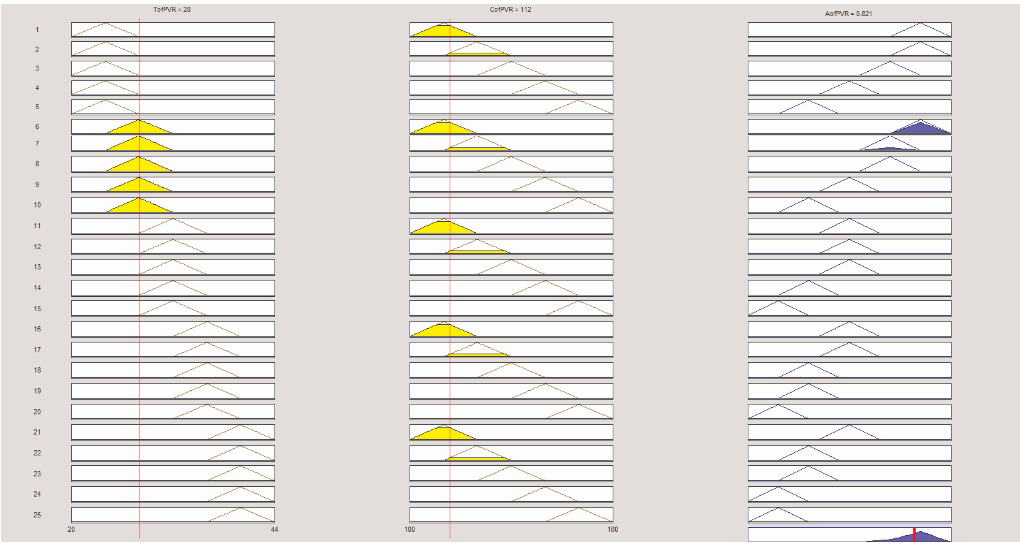

Fig. 19. Applying twenty five fuzzy rules and the output variable

5. CONCLUSIONS

The proposed approach combines in one model precise and imprecise input data. The considered activity includes hard and soft links, i.e. takes into account three types of soft links between activities in network project model introduced by El-sersy [12]. The activity costs are assumed to be considered as fuzzy numbers. Assumed framework seems to be very attractive in cases the project realisation can change.

Proposed method of project planning was implemented in MATLAB system. The employed fuzzy rules, i.e., the encompassing Min Interfering concept have been examined. Table 8 presents the comparison of the proposed method and the arbitrarily chosen commercially available methods of project planning.

Tab. 8. Comparison of the proposed method and the arbitrarily chosen methods of project planning

Scope	**CPM**	**PERT**	**GERT**	**FCPM**	**FPERT**	**SOFTCPM**	**Proposed method**
Precise time of activity/project	yes	no	yes	no	no	yes	yes
Precise cost of activity/project	no	no	no	no	no	no	yes
Imprecise time of activity/project	no	yes	no	yes	yes	no	no
Imprecise cost of activity/project	no	no	no	no	no	no	yes
Precedence relationship	hard	hard	alternative path	hard	hard	hard and soft	hard and soft
Multicriteria analysis	no	no	no	no	no	no	yes

References

[1] BOCEWICZ G., BACH-DĄBROWSKA I., BANASZAK Z.: *Declarative approach to computer aided project planning, EXIT, Warsaw, 2009*

[2] CHANAS S., KAMBUROWSKI J.: *The use of fuzzy variables in PERT*, Fuzzy Sets and Systems. 1981, 5, s. 11-19

[3] CHEN S.-P.: *Analysis of critical paths in a project network with fuzzy activity times*, European Journal of Operational Research. 2007, 183, s. 442-459

[4] CHEN C.-T., HUANG S.-F.: Appying fuzzy method for measuring criticality in project network, Information Sciencies. 2007, 177, s. 2448-2458

[5] DHAR V., STEIN R.: *Intelligent decision support methods*, The Science of Knowledge Work. Prentice Hall, New York, 2000

[6] DWEIRI F.T., KABLAN M.M.: *Using fuzzy decision making for the evaluation of the project management interval efficiency*, Decision Support Systems. 2006, 42, p. 712-726

[7] JĘDRZEJCZYK Z., KUKUŁA K., SKRZYPEK J., WALKOSZ A.: *Operational research in examples and works*, PWN, Warsaw, 2004

[8] KUCHTA D.: *Use of fuzzy numbers in project risk (criticality) assessment*, International Journal of Project Management. 2001, 19, s. 305-310

[9] MADAUSS B.J.: *Handbuch Projektmanagement*, Schäffer-Poeschel Verlang, Stuttgart, 1994
[10] TAMIMI S., DIEKMANN J.: *Soft logic in network analysis*, Journal of Computing in Civil Engineering, Vol. 2, No. 3, 1988, 289-300
[11] TROCKI M., GRUCZA B., OGONEK K.: *Project management*, PWE, Warsaw, 2003
[12] WANG W.C.: *Impact of soft logic on the probabilistic duration of construction project,* Iterantional Journal of Project management, No. 23, 2005, 600-610
[13] YOON K.P.: *A probabilistic approach to rank complex fuzzy numbers,* Fuzzy Sets and Systems. 1996, 80, s. 167-176
[14] ZADEH L.A.: *Fuzzy sets*, Information and Control. 1965, 8, s. 338-353
[15] ZARE H.K., GHOMI S.M.T.F., KARIMI B.: *Developing a heuristic algorithm for order production planning using network models under uncertainty conditions*, Applied Mathematics and Computation, 2006, 182, s. 1208-1218
[16] ZHANG H., TAM C.M., LI H.*: Modelling uncertain activity duration by fuzzy number and dicrete-event simulation,* European Journal of Operational Research. 2005, 164, s. 715-728
[17] http://www.control-systems-principles.co.uk/whitepapers/fuzzy-logic-systems.pdf, 25.02.2010

APPENDIX

The term fuzzy logic originated in the early 1965s when Lofti Zadeh [14] introduced the concept of a fuzzy set [5]. Fuzzy set theory provides a good mathematical methodology to describe and handle the problem of imprecise project cost. Fuzzy sets are used to define uncertainty of project cost. Fuzzy set theory has been applied to network-based planning techniques by some scholars such as Chanas and Kamburowski [2], Kuchta [8], Zare, Ghomi and Karimi [15], Zhang, Tam and Li [16], Chen and Hsueh [4] and others. In the network techniques with fuzzy time (Fuzzy Critical Path Method FCPM, Fuzzy Program Evaluation and Review Technique FPERT), the only fuzzy parameter is the activity time, total duration time, but others parameters like costs of activities, cost of project are certain.

The method considered is based on Work Breakdown Structure (WBS). Among others, quantities of the necessary material, man-hours of labour, equipment are determined for particular activities of the project (Table 2). Uncertain character of costs of accomplishment of particular activities is presented with the use of fuzzy sets.

Fuzzy numbers are the fuzzy sets that are normalized and convex. Accordingly triangular fuzzy sets can be called fuzzy numbers [16].

An uncertain cost of each activity can be described by a fuzzy number defined by its membership function $\mu_{\tilde{K}}(x)$, associated with numerical value in the interval of [0, 1], i.e. $\tilde{K} = \{(x, \mu_{\tilde{K}}(x)); x \in X, \mu_{\tilde{K}}(x) \in [0,1]\}$. The fuzzy number $\tilde{K}$ is a fuzzy set whose membership function $\mu_{\tilde{K}}(x)$ satisfies the following conditions [4]:

1. $\mu_{\tilde{K}}(x)$ is piecewise continuous.
2. $\mu_{\tilde{K}}(x)$ is a convex fuzzy subset.

3. $\mu_{\tilde{K}}(x)$ is the normality of a fuzzy subset, implying that for at least one element x_0 the membership grade must be 1, i.e. $\mu_{\tilde{K}}(x_0) = 1$.

Tab. 2. Specification for Statement of Work

Item	Work description Description of elements of a building	Unit	Quantity
	1. RAW CONDITION		
	1.1. Earthwork		
1	Removing up to 15 cm of layer of cultivated land with the use of a bulldozer	m^2	144,64
2	Diggings and drifts done by push shovels with a dump, a single shovel of 0,40m3, group III	m^3	52,723
3	Covering recesses by hand behind the walls of water engineering plants while making an embankment of up to 4m in soil cat. IV	m^3	18,955
4	Measurements at foundation diggings in flat and low-lying areas	m^3	71,678

It is obvious that each decision maker may have another fuzzy number related to each attribute [8]. This fuzzy number may have the shape function defined on various domains. There are many types of membership function, for example: trapezoidal membership function, Gaussian membership function, bell membership function.

In this paper, for practicality and ease of the project cost calculations, we propose triangular fuzzy numbers for the estimations. Fig. 20 shows the membership function of a triangular fuzzy number.

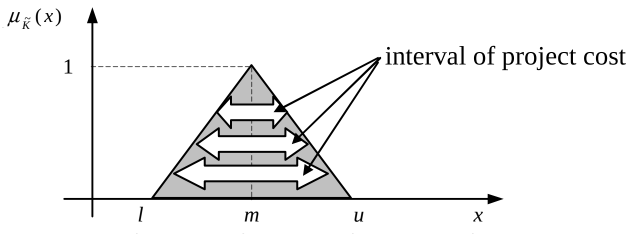

Fig. 20. Membership function of positive triangular fuzzy number

Considered membership function is defined following:

$$\mu_{\tilde{K}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \le x \le m \text{ and } l \ne m, \\ \dfrac{u-x}{u-m}, & m \le x \le u \text{ and } m \ne u, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Costs of each activity in Work Breakdown Structure are represented by triangle fuzzy numbers (pessimistic, probable, optimistic). Triangular fuzzy sets are very often used in the practice because the parameters defining them can be easily specified in linguistic terms [13]. Therefore, triangular fuzzy sets are applied to describe uncertain cost of activities.

Input parameters cost estimations are positive triangular fuzzy numbers (fuzzy cost of activities of the project network). Output parameter, i.e. total project cost is also a triangular fuzzy number – interval. The interval indicates that the project cost is estimated at this interval with α-confidence (α-cut). The α-confidence of fuzzy number of project cost $\tilde{K}$ is defined as follows [3]:

$$\tilde{K}_\alpha = \{x \in X : \mu_{\tilde{K}}(x) \geq \alpha\}, \quad \forall_{\alpha \in [0,1]}, \tag{3}$$

Note that $\tilde{K}_\alpha$ is a crisp set rather than a fuzzy set. Using α-confidence, $\tilde{K}_\alpha$ can be represented by different levels of confidence intervals. Fig. 6 shows the shape of fuzzy number with α-confidence level.

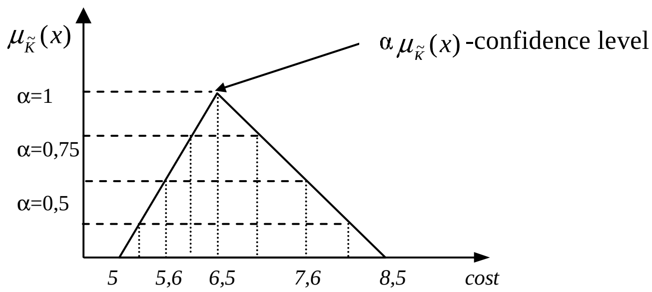

Fig. 21. The membership function of given fuzzy cost $\tilde{T}_\alpha = (5; 6{,}5; 8{,}5)$ and its α-cut

In order to reduce uncertainty of decision making (in this case determining costs of realization of particular activities of an undertaking) it is suggested that experts take part in the process. Experts's opinions may be useful in determining each cost of activity. Experts establish estimated costs of completing particular activities constituting a given project. On the basis of their knowledge and experience, the experts define costs of completing an activity with the use of fuzzy numbers. Exemplary evaluations of costs done by three independent experts are presented in Fig. 22.

Experts' opinion may be different. For the experts' ideas to be convergent, fuzzy Delphi approach can be used [15]. We proposed a simple method using the average of experts' ideas. We calculate the average of the fuzzy numbers as follows:

$$\mu_{\tilde{K}_{average}} = \left(\frac{\sum_{i=1}^{n} l_i}{n}, \frac{\sum_{i=1}^{n} m_i}{n}, \frac{\sum_{i=1}^{n} u_i}{n}, \right) \tag{3}$$

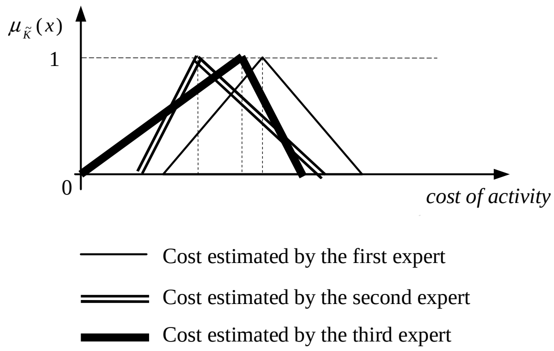

Fig. 22. Cost estimation done by experts with using fuzzy numbers